ENVESTNET PMC

Capital Markets Assumptions Methodology

Q | R | G
Quantitative Research Group

I. Overview

Capital markets assumptions are the expected returns[1], standard deviations, and correlation estimates that represent the long-term risk/return forecasts for various asset classes. We use these values to score portfolio risk, assist advisors in portfolio construction, construct our own asset allocation models and create Monte Carlo simulation inputs for portfolio wealth forecasts.

Our approach to estimating capital markets assumptions and constructing asset allocation models is based on the following general assumptions:

- The global capital markets are largely efficient in the long run, where the efficiency of the markets is measured by the Capital Asset Pricing Model (CAPM) (see Glossary of Terms).
- While the global capital markets are efficient in the long run, there might exist identifiable shorter-term inefficiencies in the capital markets.
- Risk premia are time-varying.

Our capital market assumptions construction process is based on using statistically advanced techniques to combine information coming from three sources: theory, researcher views (e.g., forecasts by recognized economic analysts or our own views into future returns of equity and fixed income asset classes), and historical data. The process consists of the steps that are detailed in the next section.

II. Process

STEP 1: Estimating Standard Deviations and Correlations

We employ a method created by Robert Stambaugh[2] (1997) to calculate standard deviations and correlations that are forward looking, in that they account for estimation risk (See the Glossary of Terms). In addition, this estimation method eliminates the need to look at only the common data periods when estimating the standard deviations and correlations—a common, but a very restrictive way to guarantee that correlation matrixes are positive definite[3]—and allows for the usage of all the available data deemed appropriate for a particular asset class.

The reasoning behind this methodology is to use the direction and strength of relationships across various asset classes for the common time periods to infer what these relationships would have been for the time periods, where one of the asset classes does not have data. In the original methodology all the available assets are used to construct these cross-asset relationships. We have improved this methodology by utilizing stepwise-fit multivariate linear regression framework (see Glossary of Terms) to guide us in estimating these cross-asset relationships.

STEP 2. Theoretical Model: Reverse Optimization

To obtain the long-term expected return estimates as implied by the CAPM, we use the reverse optimization approach proposed by William Sharpe (1974). While this approach is based on the same theoretical principles as the CAPM, it allows us to avoid estimating the risk premium on the market portfolio. Estimating the risk premium on the market portfolio can be a challenging task due to the dependence of this estimate on the data period used. Instead, the reverse optimization calls for using (A) the observed market portfolio, (B) market risk aversion coefficient, and (C) the standard deviations and correlations (estimated in Step 1), to obtain the estimates of the expected returns. These expected returns, when used in conjunction with the standard deviation and covariance estimates, then imply the observed market portfolios as the efficient market portfolio under the CAMP theory.

To estimate the observed market portfolio we estimate the market capitalizations of all the non-overlapping indexes commonly used in constructing long-only strategic portfolios (see Figure 1 for an example of such a portfolio). For example, to estimate the market capitalization of domestic equity we look at the market capitalization of Russell's Top 200 Value/Growth, Russell MidCap Value/Growth, and Russell 2000 Value/Growth indexes.

The risk aversion coefficient can be thought of as a "magnitude of the trade-off between expected return and variance" (Sharpe, 1974). Instead of trying to estimate this value, we will set this parameter to a value that makes the rate of return on domestic equity (proxied by Russell 3000 Index) implied by the reverse optimization equal to the forecast that we make in Step 3.

[1] The expected returns are given in nominal arithmetic mean terms, although as we note later the translation between nominal vs real and arithmetic vs geometric mean returns is straightforward.
[2] Robert Stambaugh is a professor of finance at The University of Pennsylvania Wharton School.
[3] See the Glossary of Terms. Also, note that positive definiteness of correlation matrixes is essential when this correlation matrix is used in optimization or simulation. Note that a correlation matrix that is obtained from individual pairwise correlations cannot be guaranteed to be positive definite.

Thus, the reverse optimization framework can be thought as a way of obtaining the correct relative expected return relationships among various assets, while the methodology in Steps 3 and 4 (i.e., obtaining of Researcher Views) guides us in setting the levels of these forecasted expected returns.

STEP 3. Researcher Views: Equity

We forecast the return for the Russell 3000 Index (which proxies for the entire domestic equity asset class) and use this estimate as an anchor for the expected return levels for the other asset classes in Step 2.

Any equity return (both realized and expected) can be broken down into parts that are attributable to dividend yield and capital gains. Capital gains can be further broken down into a portion that is attributable to the growth in earnings per share and a portion that is attributable to growth in P/E ratios. These are exact algebraic relationships, and if viewed independently of each other do not provide any additional insight for purposes of forecasting. However, if we assume that pricing multiples (e.g., P/E's) are mean-reverting (or at least are not likely to stray orders of magnitude outside historical norms), then, as shown in a seminal paper by Campbell and Shiller (1988), present dividend ratios have to forecast either future increases in earnings per share or decreases in future returns. In other words, with this dynamic relationship in place, we can start tying the three components of the return (dividend yield, earnings per share growth, and change in P/E ratios) to each other and to the current market information (e.g., current price multiples and earnings per share growth expectations).

Forecasting the Dividend Yield.

As noted by Campbell and Viceira (2005) as well as Ang and Bekaert (2006), dividend yields follow relationships that are almost random walks, which means that the best prediction for a future dividend yield is today's dividend yield. Hence, we use the current dividend yield on Russell 3000 index as an estimate for the dividend income part of the nominal geometric return estimate.

Forecasting the Growth in Earnings Per Share.

The nominal growth rate of earnings can be broken down in two pieces—the expected inflation and the growth rate of real earnings. As noted in Ilmanen (2011), it is often mistakenly assumed that the rate of real GDP growth is a good proxy for the growth rate of real earnings per share. However, a much better proxy both empirically and also intuitively is the rate of real GDP growth per capita, which under positive population growth scenario is usually substantially lower than the headline real GDP growth rate. Since after the World War II, the real GDP growth has been almost 3 percent, while the real GDP growth per capita has been only slightly above 2 percent. In fact, there have never been prolonged periods with above 2 percent real GDP per capital growth rates outside of the 1990s, when it averaged 12 percent.

Arnott (2011) notes that while the aggregate real earnings track the real GDP growth well, the real earnings per share grow at a rate that is significantly slower than the aggregate real earnings, mainly due to a dilution effect. That is, a large part of aggregate earnings growth happens due to growth in new business, which is not reflected in the existing stock market indexes.

Forecasting the Change in P/E.

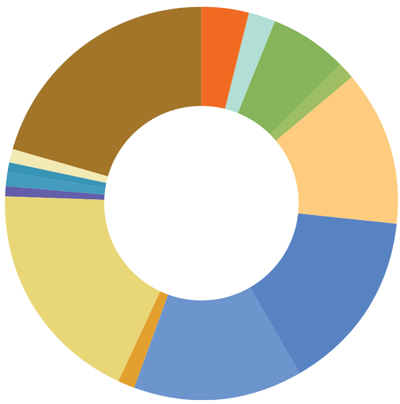
If the P/E's are mean reverting, then today's P/E's carry information either about future growth of earnings per share or future returns, or both (Campbell and Shiller, 1988). In addition, as shown in Campbell and Shiller (1998), current P/E's have a strong negative correlation with future returns, while at the same time they have practically no correlation with the future earnings per share. With the above dynamic relationship in mind, high/low levels of current P/E's can be expected to correlate to low/high future rates of equity returns (the most likely mechanism is through multiple repricing) and have negligible forecasting power for the change in real per share earnings.

Thus, to estimate the part of the return that comes from P/E's mean reverting, we look at the current level of Russell 3000 Index P/E and calculate the annual rate of return over the next 10 years that will be added to/ subtracted from the return while the current P/E moves to its long-term mean or "anchor". For various reasons (see Asness, 2011), P/E multiples from distant past are not very relevant for calculating this anchor. Rather, we form this anchor as a weighted average of the current level of P/E's and P/E's going back to early 1970's, where the period since 1970's serves as a "long-term" horizon.

The reason that we use current level of P/E's as one of the components in our P/E anchor calculation is to account for the possibility that the reversion to this anchor from the current P/E levels happens very gradually over time.

Figure 1

2021 World Portfolio



Cash	4.04%
Commodities	2.19%
Emerging Markets	6.41%
High Yield	1.30%
Int'l Developed Equity	12.86%
Large Cap Growth	14.95%
Large Cap Value	13.93%
Munis	1.36%
Non-US Fixed Income	18.60%
Real Estate	0.93%
Small Cap Growth	1.04%
Small Cap Value	0.96%
TIPS	1.02%
US Fixed Income	20.42%
Grand Total	**100.00%**

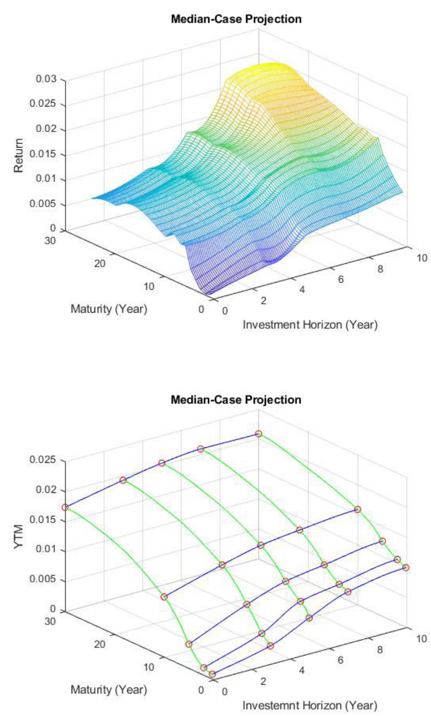
Source: Envestnet | QRG estimate.

On the other hand, the reason for choosing early 1970's as the starting point of our long-term horizon is that we believe financial markets across the world experienced a structural break in 1971, as United States unilaterally withdrew from the Bretton Woods monetary system, effectively causing it to collapse. Because of the breakdown of this system, which essentially allowed the exchange rates of major economic powers to float freely against each other, the central banking authorities were free to engage in inflationary monetary policies, which they subsequently did. We believe that this ability on behalf of the central banks to engage in largely unchecked expansionary monetary policies is one of the reasons that the P/E's have been on an upward trend ever since the beginning of 1970's with only brief intermittent pauses and reversals.

Finally, to construct a level of the P/E anchor, where the base case scenario consists of P/E's slowly adjusting towards their long-term anchor from their current levels, we assign a weight of 70 percent to the current level of P/E's and 30 percent to the historical level (since early 1970's) level of P/E's.

The last step in estimating the nominal arithmetic rate of return for Russell 3000 Index is to convert the nominal geometric rate of return to the arithmetic rate of return by adding to it half of its variance.

Figure 2



To summarize, the forecast of the nominal arithmetic expected return consists of the following components:

- Forecasted dividend yield, proxied, as explained earlier, by the current dividend yield.
- **Expected Growth Rate in Real Earnings Per Share:** As mentioned earlier, the real GDP growth rate has historically formed a ceiling on real earnings per share growth rate, which is much better tracked by the growth rate of real GDP per capita and lower than the real GDP growth rate. To allow for a possibility of above-average real earnings per share growth rate, we assume that it will track the real GDP growth rate over the next decade.
  To obtain the forecast for the real GDP growth rate, we survey various professional forecasting sources, including: the Congressional Budget Office (CBO), the Federal Open Market Committee (FOMC) Consensus Forecast, a survey of professional forecasters (Philadelphia Fed), the Social Security Administration's Trustee Report, the IMF World Economic Outlook, the World Bank, and others.
- **Forecasted Inflation:** To obtain the inflation forecast, we survey the same professional forecasting sources used to determine the average real GDP growth rate. In addition, we also consider market information: the yield spreads between the 10-year nominal Treasury notes and the inflation-adjusted fixed income securities (TIPS).
- **Valuation Multiple Adjustment Factor:** This component represents the gradual valuation multiple adjustment back to their long-term values. It adds a positive contribution, if the current multiple is below the historical anchor and a negative, if the opposite is true.
- **Geometric-to-Arithmetic Rate Conversion:** To convert the return from geometric to arithmetic units, we add half of the variance of the asset class to the geometric return.

STEP 4. Researcher Views: Fixed Income

The fixed income expected return forecasting methodology was developed for the purposes of informing our Black-Litterman views for the fixed income asset classes (see Envestnet's white-paper "Forecasting Constant Maturity Bond Returns"). The crux of the methodology is to translate yield-to-maturity forecasts into returns on constant maturity bond portfolios. The methodology serves as an interface between the yield-to-maturities, which are usually the object of forecasts, and the returns on constant maturity portfolios, which are the values that the investors are ultimately interested in. Since the constant maturity bond returns depend on coupon payments as well as capital gains, which move in opposite directions when the yield curve shifts, the calculation of the forecasted total return requires careful mathematical modeling and analysis.

The methodology starts out with a set of yield-to-maturity forecasts for 3 month to 30 year maturity bills and bonds, at various maturity increments, and at 5- and 10-year forecast horizons. To form our yield curve forecasts, we have to take a stance on whether the current forward rates are good forecasts on future expected yields. If we believe that forwards are good forecasts for the expected future yields, we would be taking the position of Pure Expectations Hypothesis, PEH, which implies that Bond Risk Premium, BRP, is zero. Alternatively, we can make the opposite extreme assumption, and assume that yields follow a random walk process, which means that the current yields are best predictors of the future yields, and therefore any current yield spread reflects only the required BRP rather than any market expectation for rate changes.

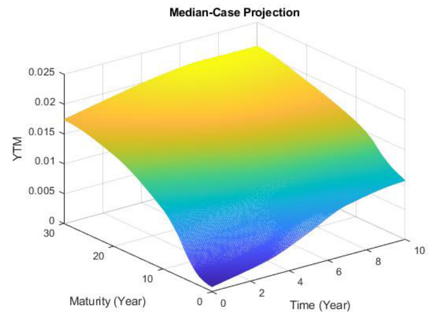
Researchers have been debating which one of these extremes fit the data best for a long time. For decades data seemed to favor the random walk hypothesis (i.e., BRP is positive and yield differences reflect mostly the risk premium that is required by the holders of longer-term maturity bonds, rather than market's expectations of yield changes), but over the last decade and a half, with BRP hovering around zero, the PEH seems to be making a comeback, as BRPs have spiked dramatically since the late summer of 2016, so it might be random walk hypothesis' time in sun.

We choose not to take a stance in this debate, but rather weigh both of these approaches equally by setting the yield curve forecasts equal to halfway between their current level (random walk hypothesis) and those implied by the forward rates (PEH).

We then use mathematical smoothing algorithms (e.g., monotone cubic spline smoothing) to fill in the yield-to-maturity forecasts for 3 month, 2-, 5-, 10-, and 30-year maturity bills and bonds at all the forecast horizons below 10 years at monthly frequency. Next, we use Nelson-Siegel method for estimating the complete yield-curve at a particular forecast horizon. These two steps allow us to translate the initial yield-to-maturity forecasts into a complete yield-to-maturity surface with monthly increments in the forest horizons and maturities (see Figure 2 for an example of such a forecasted yield surface).

Finally, using the obtained yield-to-maturity surface, we calculate total return on a hypothetical constant maturity bond portfolio. This is done at a range of maturities and investment horizons. Figure 3 gives an example of the total return surface corresponding to the yield curve surface given in Figure 2.

Figure 3



STEP 5. Putting It All Together: Black-Litterman (Bayesian) Process

To obtain our forecasts for the nominal expected rates of return, we use Black-Litterman (Black & Litterman, 1991) methodology. The Black & Litterman methodology allows only for combination of expected returns coming from reverse optimization and the views regarding the relative size of future expected returns of various asset classes. By viewing Black-Litterman methodology as a type of Bayesian approach, we have generalized the methodology to combine information that comes from the following three sources: theory (reverse optimization, obtained in Step 2), our views about the absolute size of future expected returns (Steps 3 and 4), as well as historical data.

Finally, the Black-Litterman process requires that we specify a risk-free rate of return. We use the 10-year constant maturity Treasury yield as a proxy.

STEP 6: Expected Return Forecasting for Alternative Asset Classes

To estimate the standard deviations and correlations of the new alternative asset classes, we use the approach described in Step 1. Note that this approach is particularly useful in this instance, since the data history for the alternative asset classes is relatively short, when compared to the data histories of the more traditional asset classes.

The alternative asset classes do not fit into the reverse optimization and Black-Litterman framework used for the other asset classes. This is because the strategies and the funds representing these strategies invest in the asset classes that are already represented in the calculation of the world portfolio in Step 2. Counting the market capitalization as part of the world portfolio would, therefore, result in double-counting and in artificial inflation of the world portfolio.

Because the alternative asset classes invest in the traditional asset classes and therefore cannot be neatly folded into our reverse optimization and Black-Litterman framework, we use a build-up method to estimate their expected returns. More specifically, we estimate their historical risk premia and add them back to the risk-free rate to obtain their expected return forecasts. The estimated risk premium of a particular alternative strategy (e.g., market neutral or managed futures) includes not only any inherent risk premium associated with a particular investment strategy, but also represents the average alpha of the managers in the particular alternative strategy. These two parts of the estimated risk premium are closely linked, since, unlike the traditional asset classes, the benchmarks that proxy the alternative asset classes are comprised of active managers that engage in a particular strategy. Since the active management alpha should equal to zero in aggregate, the influence of the individual manager alpha on the estimated risk premium should be minimal.

20-year Forecasts

In addition to the 10-year CMA forecasts, we also provide 20-year forecasts. The methodology between the ten and 20-year forecasts differs in the following way. First, we change our expected inflation forecast period from ten to 20-years. In particular, our forecast is guided by the spreads between 20-year Treasury and TIPS yields as well as 20-year forecasts by professional forecasting sources. Second, the valuation multiple adjustment period is assumed to be twenty, rather than ten, years. Thus, a difference between the current P/E and historical P/E is assumed to amortize over a twenty year period. Third, when developing researcher views for fixed income, we develop yield surface forecasts out to twenty year horizon, which then allows us to form constant maturity Treasury portfolio return forecasts over a twenty year horizon. Fourth, we use a twenty-year constant maturity Treasury yield as a risk-free rate in the Black-Litterman process. Finally, we use a twenty-year constant maturity Treasury yield as the risk-free rate in the alternative asset class expected return estimation.

III. References

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IV. Glossary of Terms

Bayesian statistical approach. A statistical framework that allows for consistent integration of various sources of information. A Bayesian approach allows for integration of data (e.g., returns for an asset class) with external information, such as uncertainty about the model parameters (i.e., mean, standard deviation, correlation, etc.) and other views imposed by an analyst.

Black-Litterman methodology. A Bayesian quantitative model that incorporates information coming from the following two sources: expected returns of asset classes as predicted by a theoretical model (implemented through reverse optimization) and the views of an analyst regarding the means of asset returns.

Capital Asset Pricing Model (CAPM). Used to determine a theoretically appropriate required rate of return of an asset, if that asset is to be added to an already well-diversified portfolio, given that asset's non-diversifiable risk. The model takes into account the asset's sensitivity to non-diversifiable risk (also known as systematic risk or market risk), often represented by the quantity beta (ß) in the financial industry, as well as the expected return of the market and the expected return of a theoretical risk-free asset. The CAPM was independently authored by Jack Treynor (1961, 1962), William Sharpe (1964), John Lintner (1965), and Jan Mossin (1966).

Correlation. A statistical measure of how two securities move in relation to each other. Perfect positive correlation (a coefficient of +1) implies that as one security moves, either up or down, the other security will always move in the same direction. Perfect negative correlation (a coefficient of -1) means that if one security moves up or down, the negatively correlated security will always move in the opposite direction. If two securities are uncorrelated, the movement in one security does not imply a linear movement up or down in the other security.

Estimation risk. Sometimes also called "parameter uncertainty" is the error introduced in portfolio construction process that arises from differences in the values of forecasted and realized expected returns, standard deviations, and correlations.

Expected return. The mean of a probability distribution of returns.

Mean-variance optimization. A method to select portfolio weights that provides optimal trade-off between the mean and the variance of the portfolio return for a desired level of risk.

Positive definiteness. A property of correlation matrixes that guarantees that the variance and standard deviation of any portfolio constructed using this correlation matrix will be positive.

Russell 3000 Index. An index that encompasses the 3,000 largest U.S.-traded stocks, in which the underlying companies are all incorporated in the United States. Often used as a benchmark for the entire U.S. stock market.

Standard deviation. A statistical measure of dispersion of the observed return, which depicts how widely a stock or portfolio's returns varied over a certain period of time. When a stock or portfolio has a high standard deviation, the predicted range of performance is wide, implying greater volatility.

Stepwise-fit multivariate linear regression. A systematic method for adding and removing terms in a regression model based on their statistical relevance. The technique allows for the construction of parsimonious models with robust explanatory power.

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